C&EE 141

Beam-Columns

Beam Columns

- Most structural elements are subjected to both bending and axial loads (compression or tension).
- In many cases, it is reasonable to neglect combined effects:
    - Pin-pin beam with very minor axial forces
    - Pin-pin column with only accidental eccentricities
- However, many members have significant amounts of both bending and axial load, and must be designed as “beam-columns”.

Examples of Beam-Columns

- Beams in braced frames
- "Drag" and "Chord" members
- Beams and columns in moment-resisting frames
- Columns that brace exterior wall construction for wind & seismic loading

Beam-Column Examples

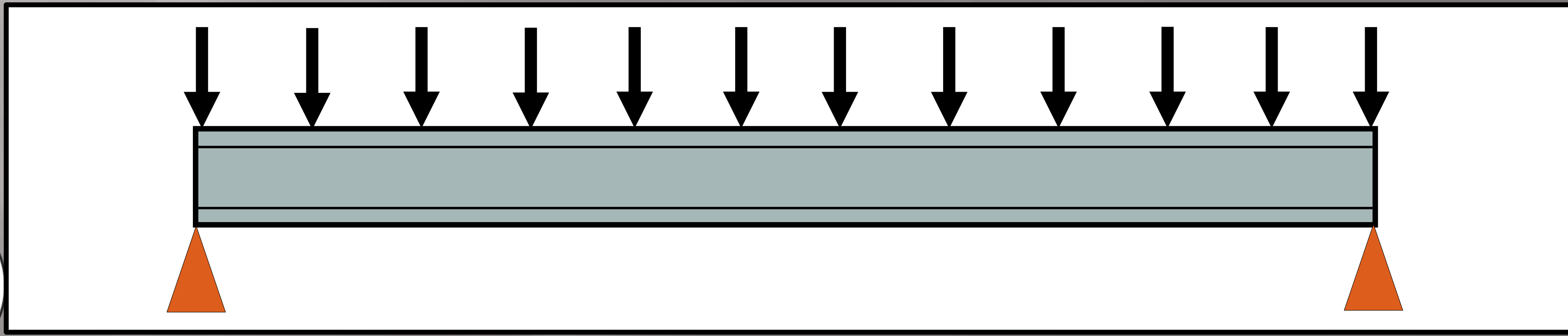
Beam-Column Examples

Second Order Analysis

Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Initially, consider a member subjected to flexure only.



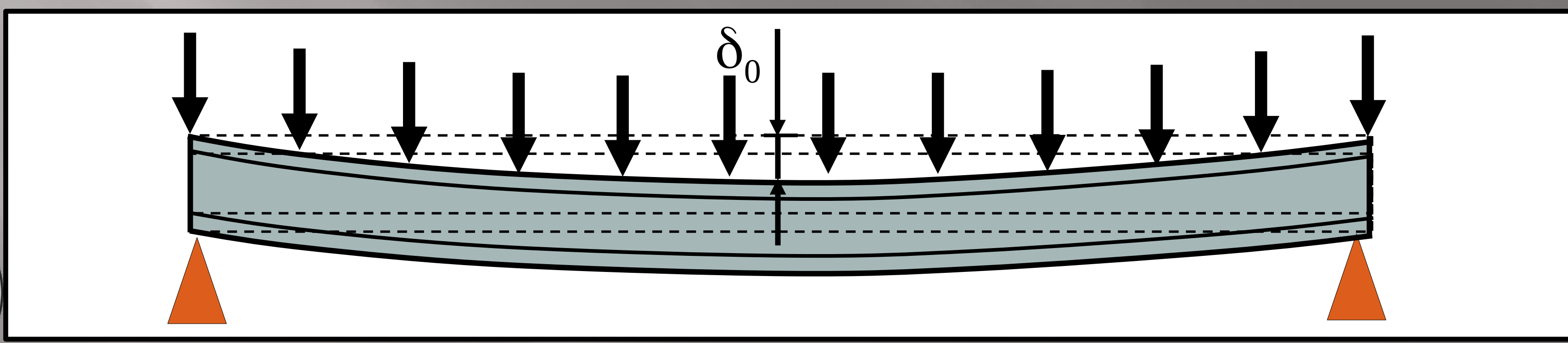
Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Initially, consider a member subjected to flexure only.

Application of load results in mid-span deflection δ_0,

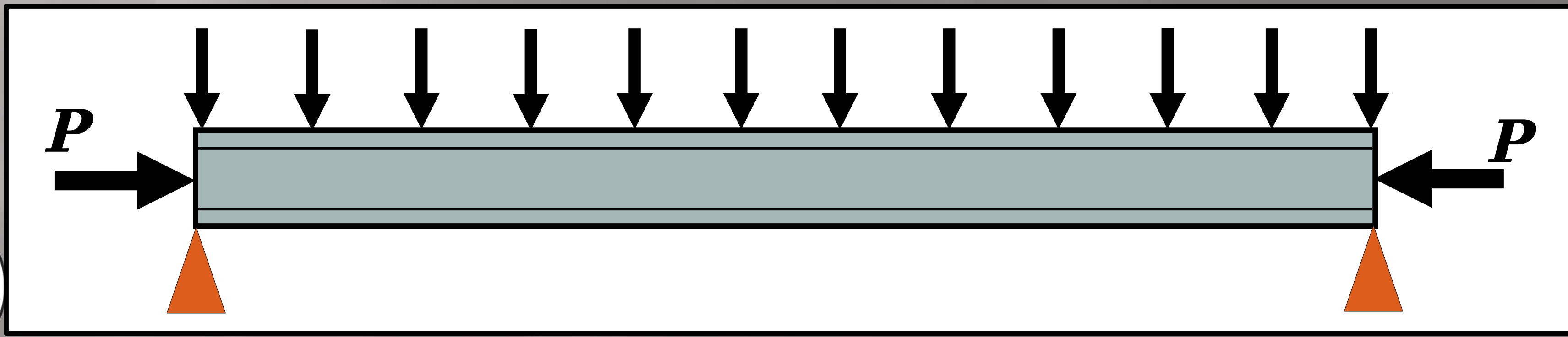
$\ddot{a}_0 = \frac{5}{384}\frac{\grave{u}L^4}{EI}$ from basic derivations.



Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

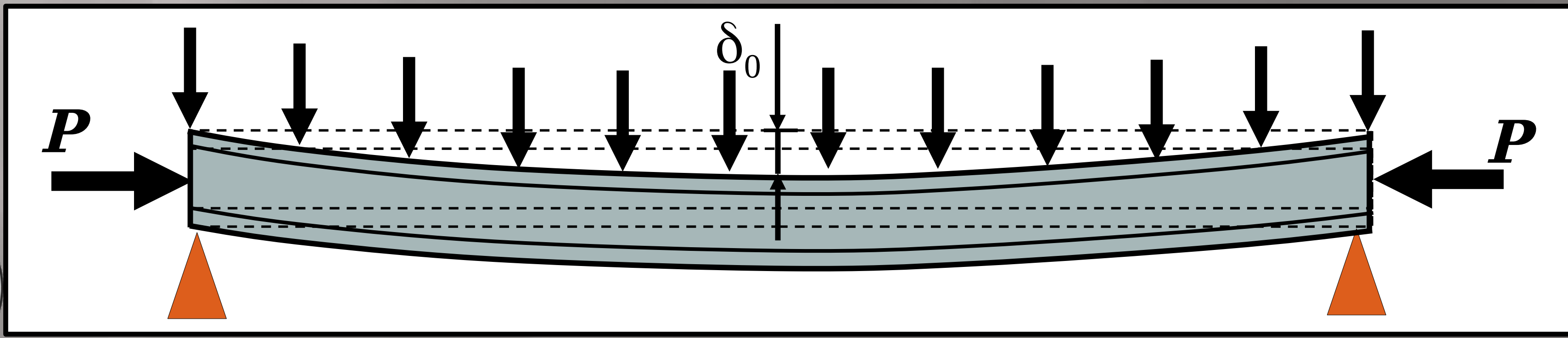
Now consider the same member with Axial load *P*.



Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Axial force acting through deformations results in additional moment $P(\delta_0)$ at center of span.

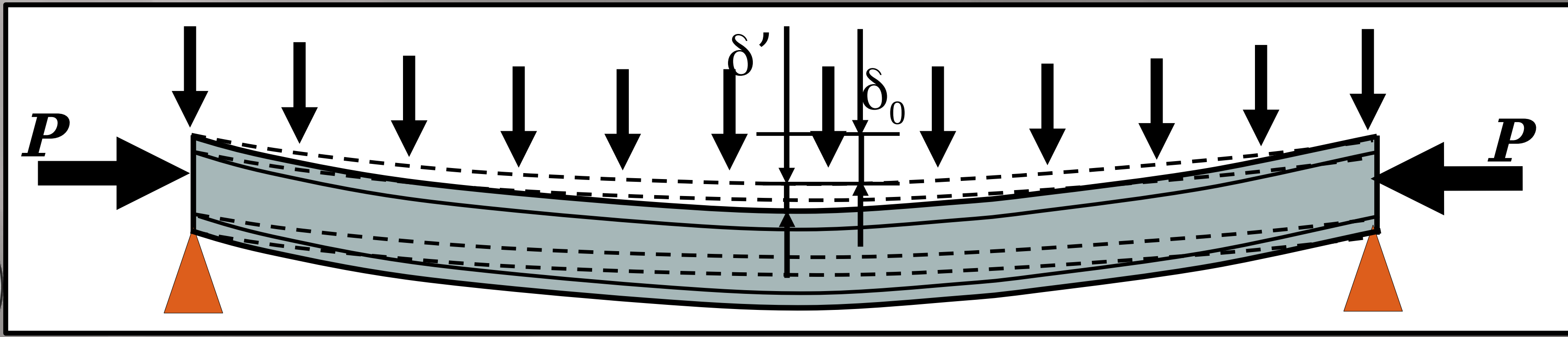


Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Axial force acting through deformations results in additional moment $P(\delta_0)$ at center span.

Additional moment then results in displacement δ'.



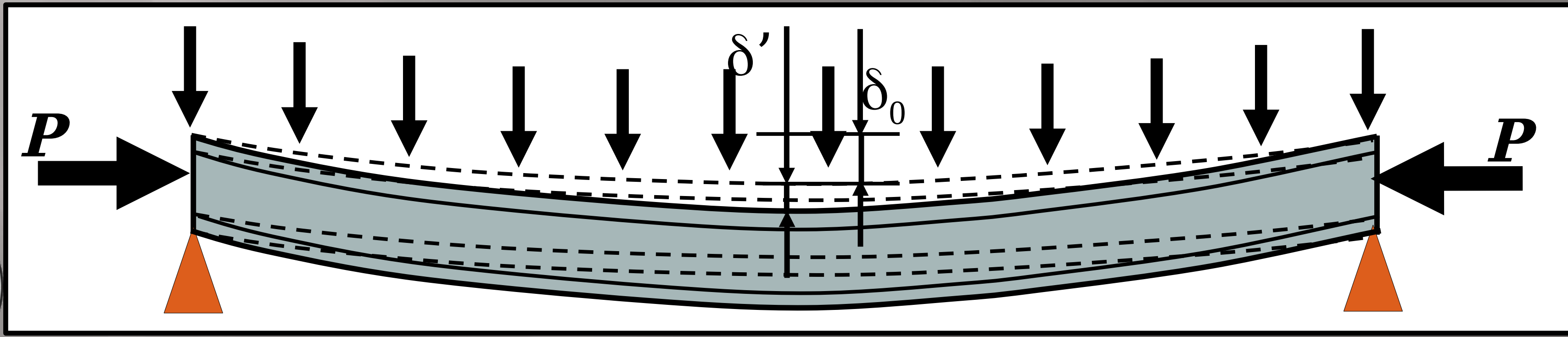
Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Axial force acting through deformations results in additional moment $P(\delta_0)$ at center span.

Additional moment then results in displacement δ'.

Resulting in additional moment $P(\delta')$



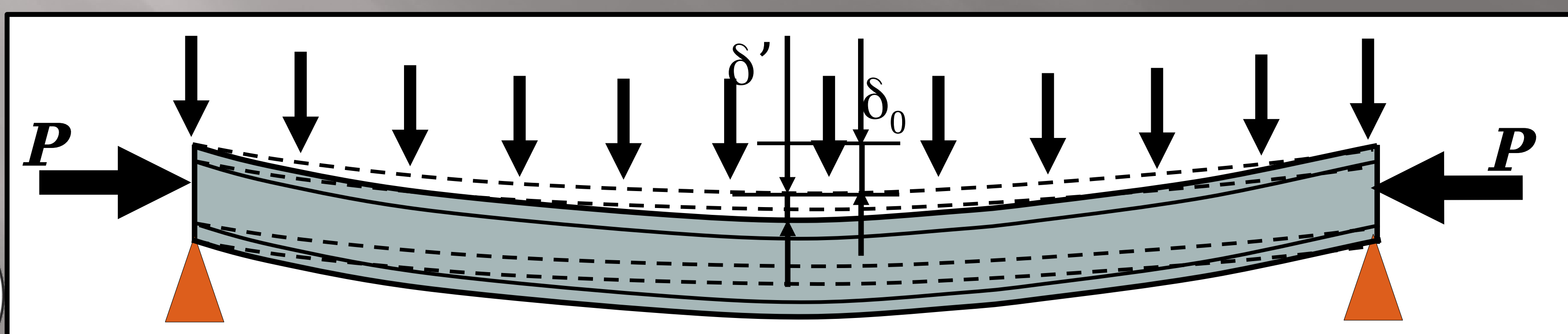
Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

Axial force acting through deformations results in additional moment $P(\delta_0)$ at center span.

Additional moment then results in displacement δ'.

Resulting in additional moment $P(\delta')$



Second Order Effects

Equilibrium is based on DEFORMED GEOMETRY.

This either results in an incremental failure, or stabilizes.

$\delta > \delta_0$ and $M > M_0$

where δ_0 and M_0 are first order results based on original geometry.

$M = M_0 + P\,\delta$

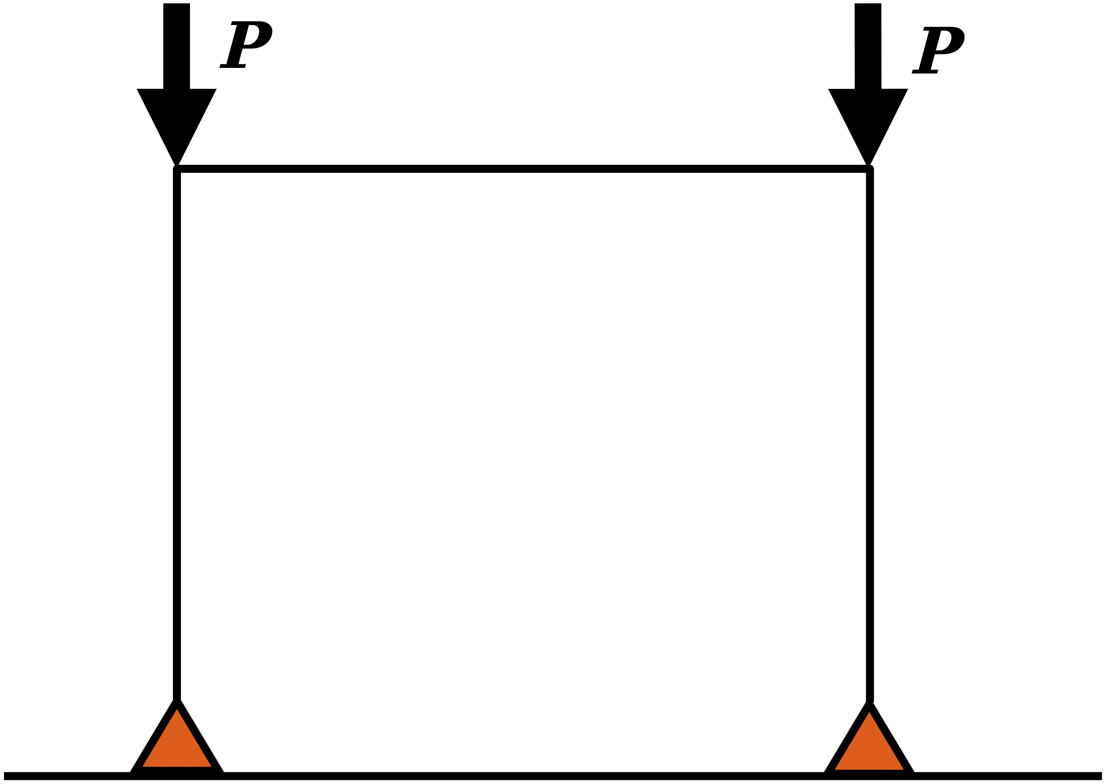
but δ depends on M…

Second Order Effects

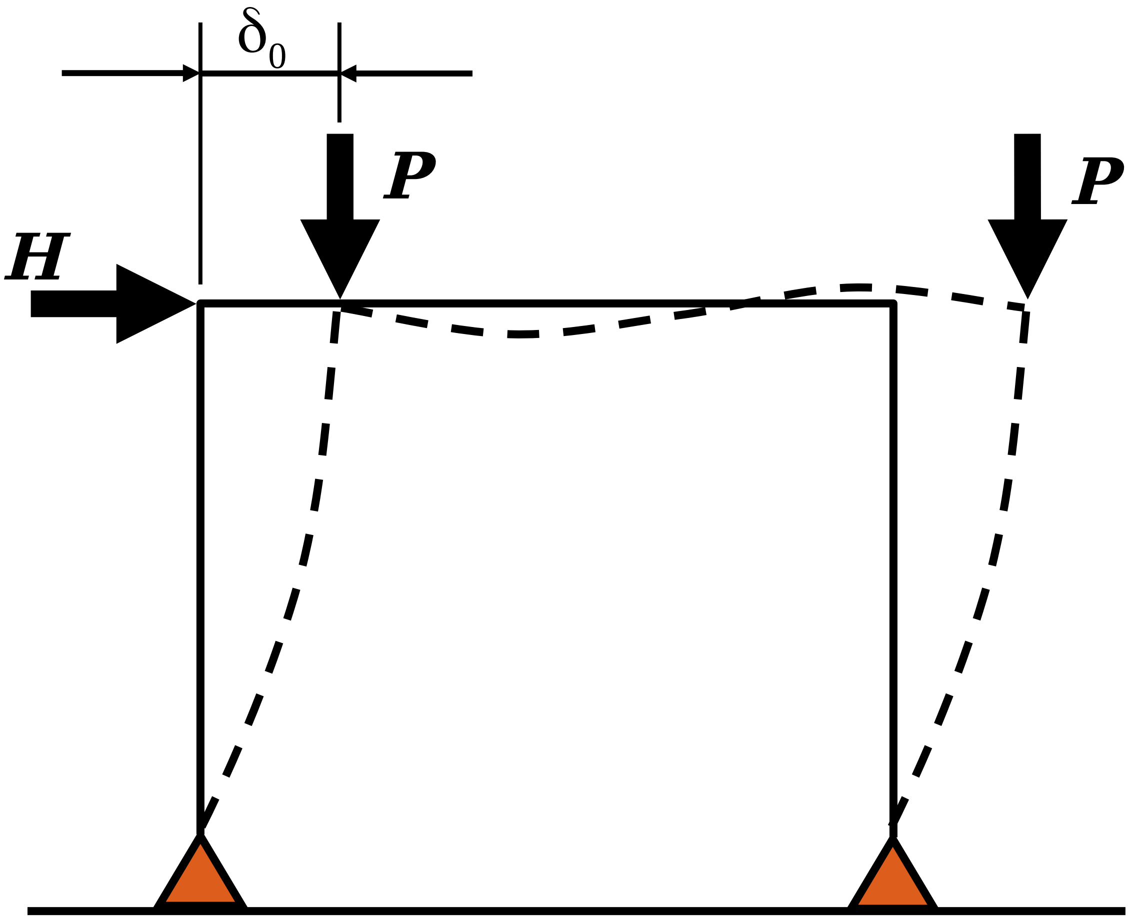
When considering a frame with loads applied at joints the same principles can be applied.

In these cases, we define joint deflections as δ, with δ_0 being the first order joint deflection.

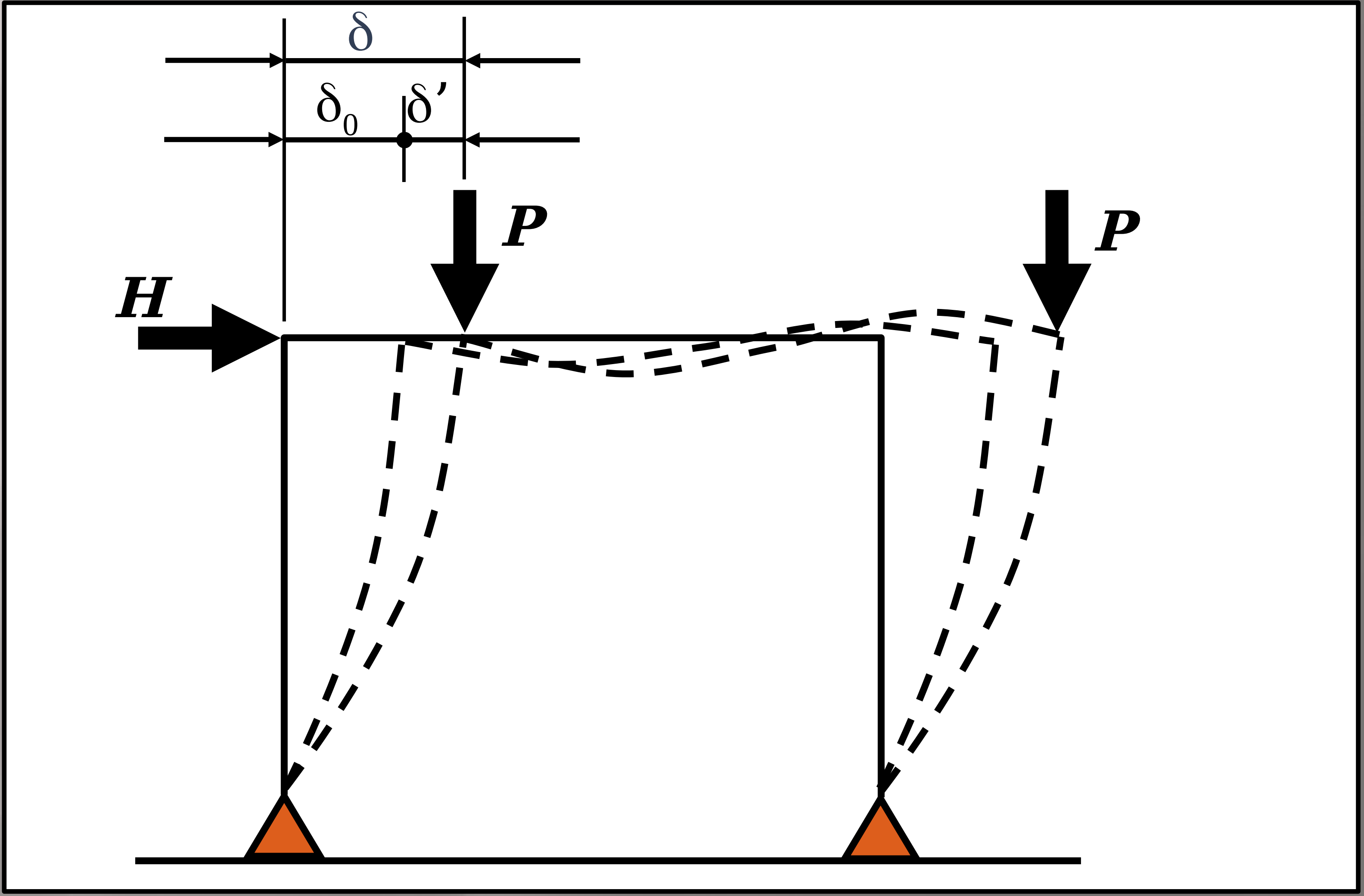
Second Order Effects



Second Order Effects

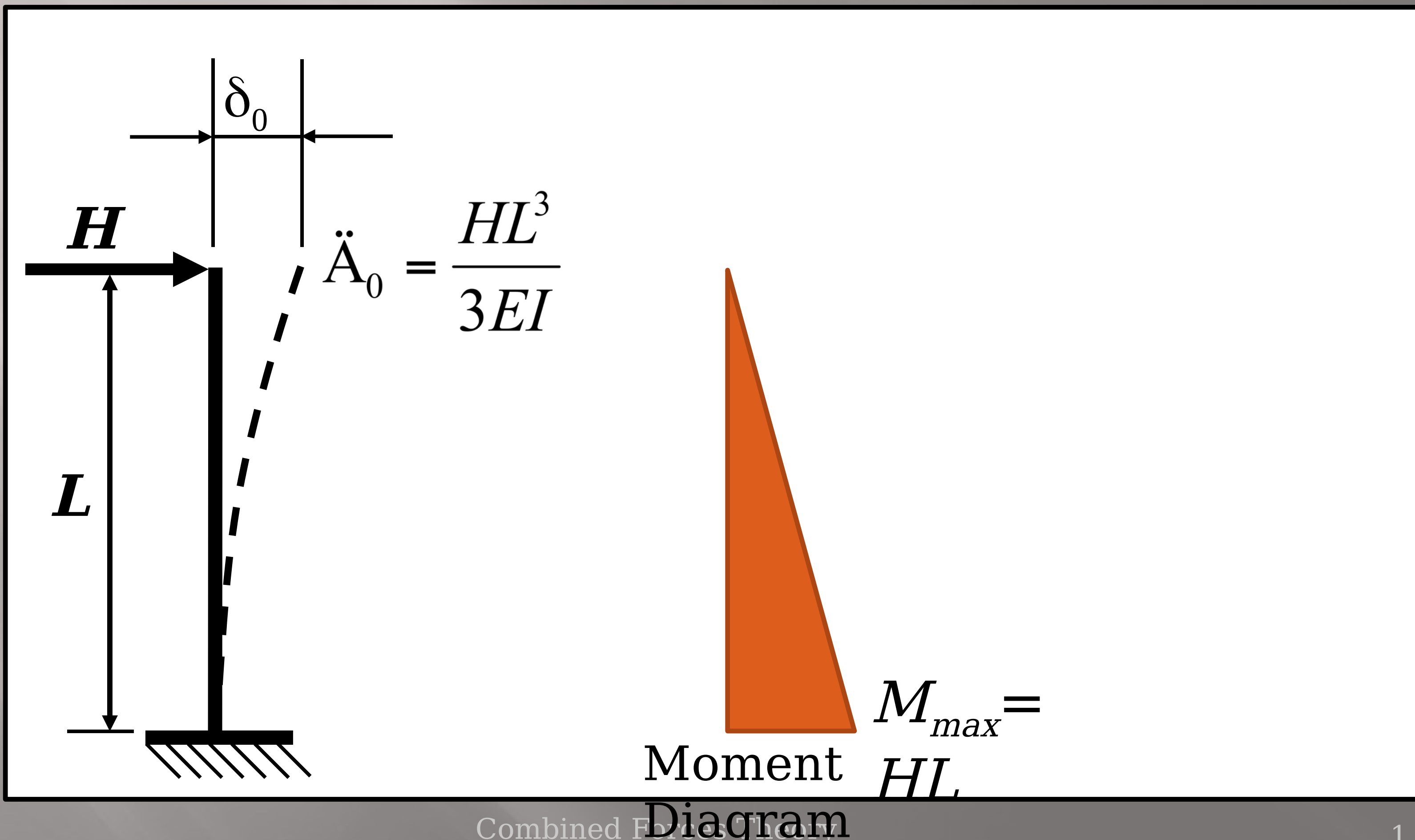


Second Order Effects



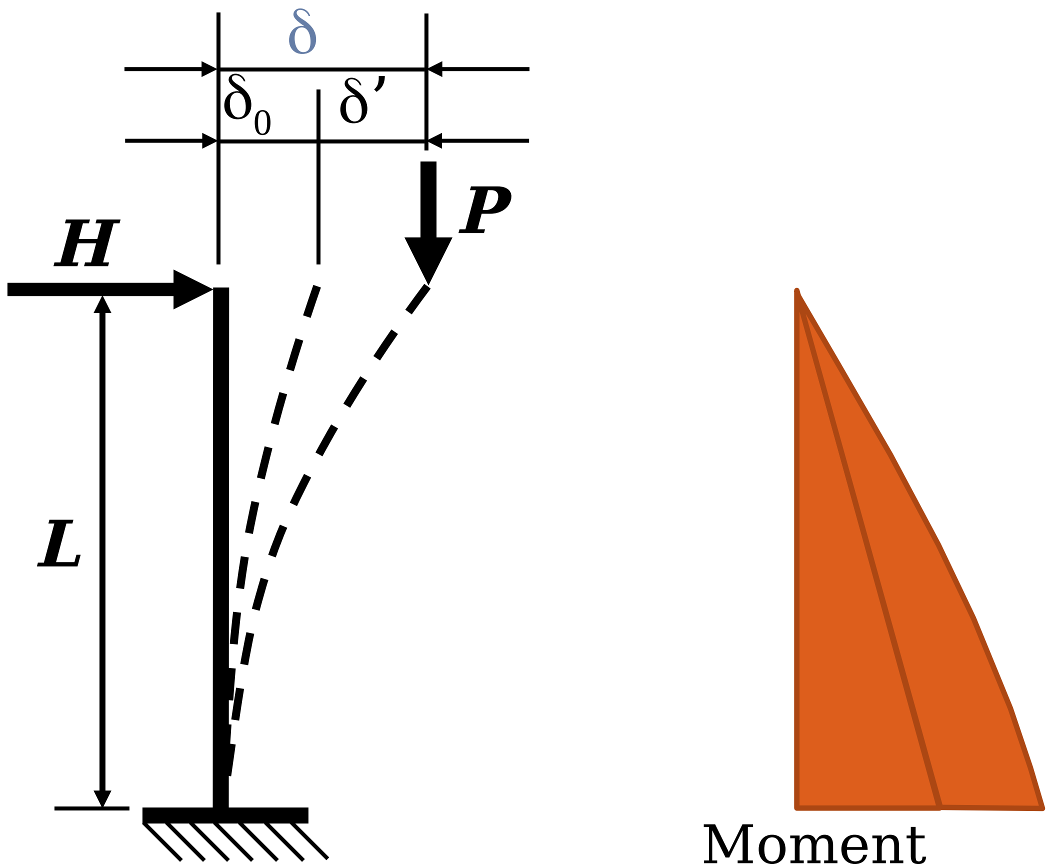
Second Order Effects

Assuming deflections are in the shape of a sine curve



Second Order Effects

Assuming deflections are in the shape of a sine curve



Moment Diagram

$M_{max} = HL + P\Delta$

Second Order Effects

Tension forces and Flexure

Tension forces on a member tend to "straighten" the member. Usually, they do not introduce 2^{nd} order effects.

Multiple states of stress are still present and need to be accounted for.

Tension in a member can also make lateral torsional buckling less likely to occur.

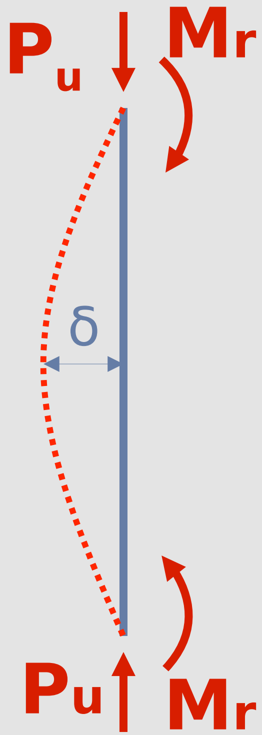
Approximate 2nd Order Analysis

- Per AISC 360-10 allows several methods to be used as alternatives to more rigorous (and tedious) 2nd Order Analysis
- Common procedures:
  - “Direct Analysis Method” (DM) per AISC Chapter C.2
  - Approximate 2nd Order Analysis per AISC Appendix 8

Approximate 2nd Order Analysis

- AISC Spec Appendix 8, Approximate 2nd Order Analysis
- Common in local practice
- Consists of 1st order elastic analysis with multipliers B_1 and B_2 applied to resulting moments ("Moment Magnification")

Moment Magnification of Columns with No Sidesway (Braced Frame)



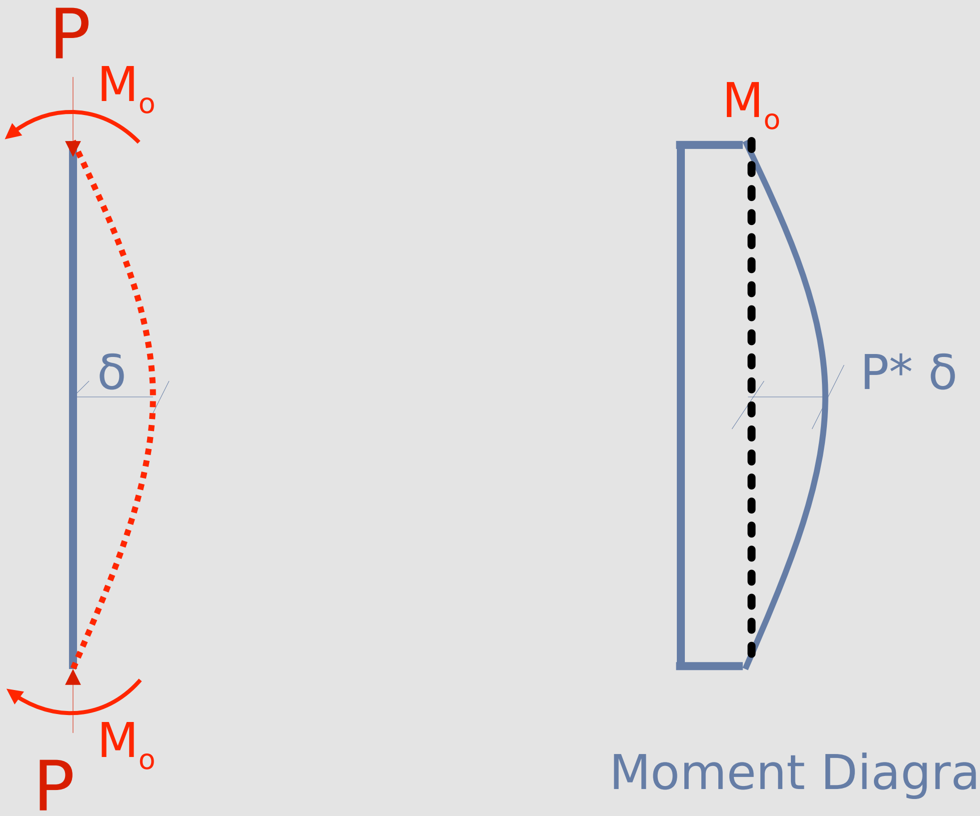
$$M_r = M_{nt} + P_u\delta = B_1 M_{nt}$$

2^{nd} Order Moment

1^{st} Order Moment

“Member Effect” per Geschwindner

B_1 Multiplier For No Sidesway

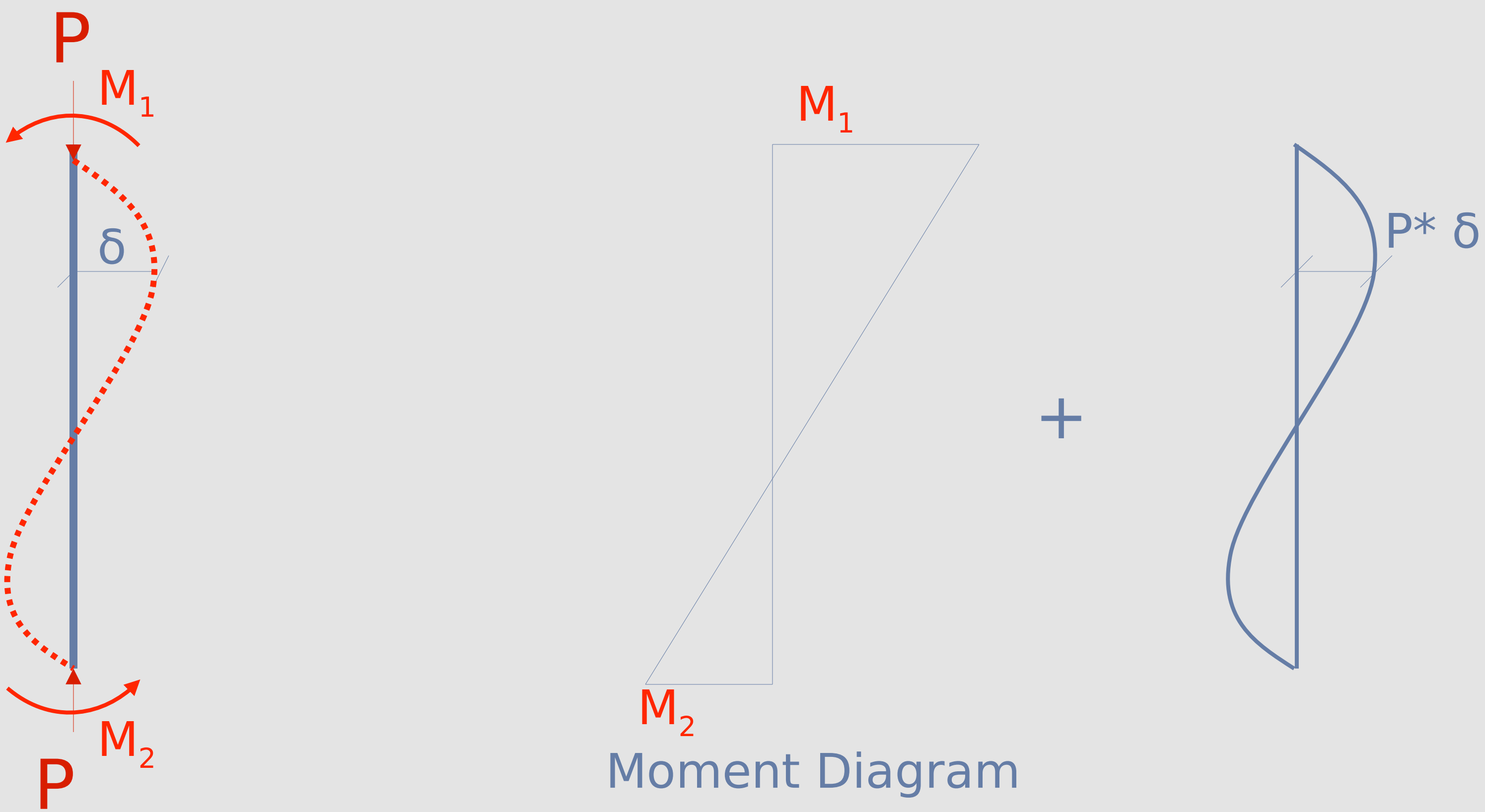


Moment Diagram

$M_{max} = M_o + P^* \delta$

End moments in opposing direction: single curvature bending. Primary and secondary moments are additive.

B_1 Multiplier For No Sidesway



End moments in same direction: <u>double curvature</u> bending. Depending on magnitude of P, M_1,M_2, and L, M_{max} may be equal to M_2 or greater than M_2.

B_1 Multiplier For No Sidesway

- The worst case will always be single curvature bending; double curvature bending tends to minimize the moment amplification.
- This effect is accounted for with a C_m factor.

B_1 Multiplier For No Sidesway

- $B_1 = C_m/(1-\alpha P_r/P_{e1}) \geq 1$
  - C_m will always be ≤ 1.0
  - $\alpha = 1.0$ for LRFD
- $P_{e1} = \pi^2 EA_g/(KL/r)^2$
  - Computed about the axis of bending.
- $C_m = 0.6 - 0.4(M_1/M_2)$
  - For no transverse loads acting on the member
  - M_1=Smaller end moment; M_2=Larger end moment
  - M_1/M_2 >0 for reverse curvature, M_1/M_2 <0 for single curvature
- See Table C-A-8.1 in AISC 360-10 for C_m factors when beam-column is loaded between joints.

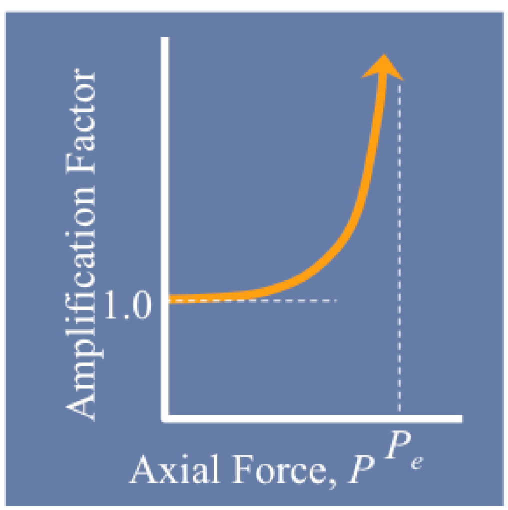
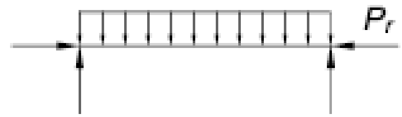
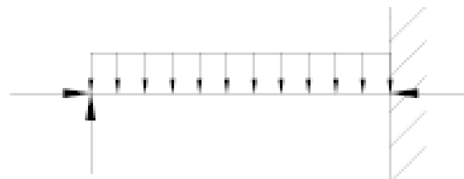
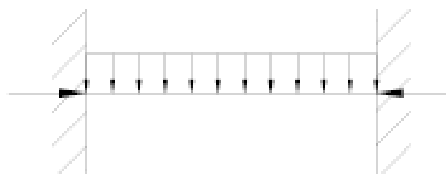
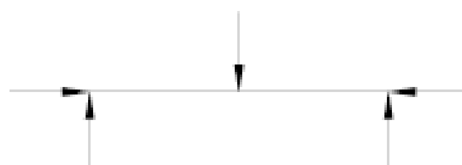
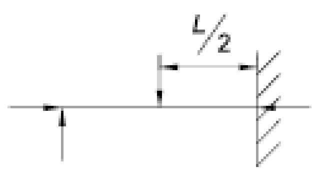
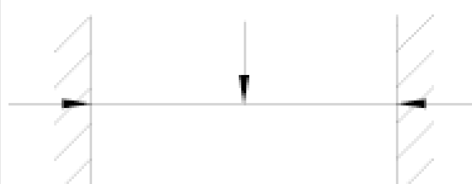
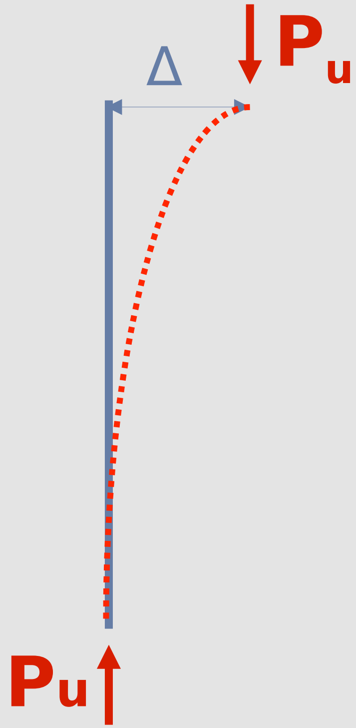


TABLE C-A-8.1
Amplification Factors ψ and C_m

Case	ψ	C_m
	0	1.0
	−0.4	$1-0.4\frac{\alpha P_r}{P_{e1}}$
	−0.4	$1-0.4\frac{\alpha P_r}{P_{e1}}$
	−0.2	$1-0.2\frac{\alpha P_r}{P_{e1}}$
	−0.3	$1-0.3\frac{\alpha P_r}{P_{e1}}$
	−0.2	$1-0.2\frac{\alpha P_r}{P_{e1}}$

Moment and Axial Magnification of Columns with Sidesway (Moment Frames)

"Structure Effect" per Geschwindner



$$M_r = M_{lt} + P_u\Delta = B_2\, M_{lt}$$

2nd Order Moment

1st Order Moment

$$P_r = P_{nt} + B_2\, P_{lt}$$

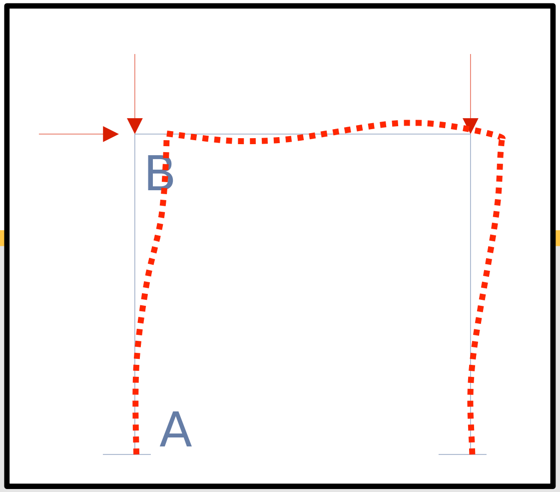
2nd Order Axial Load

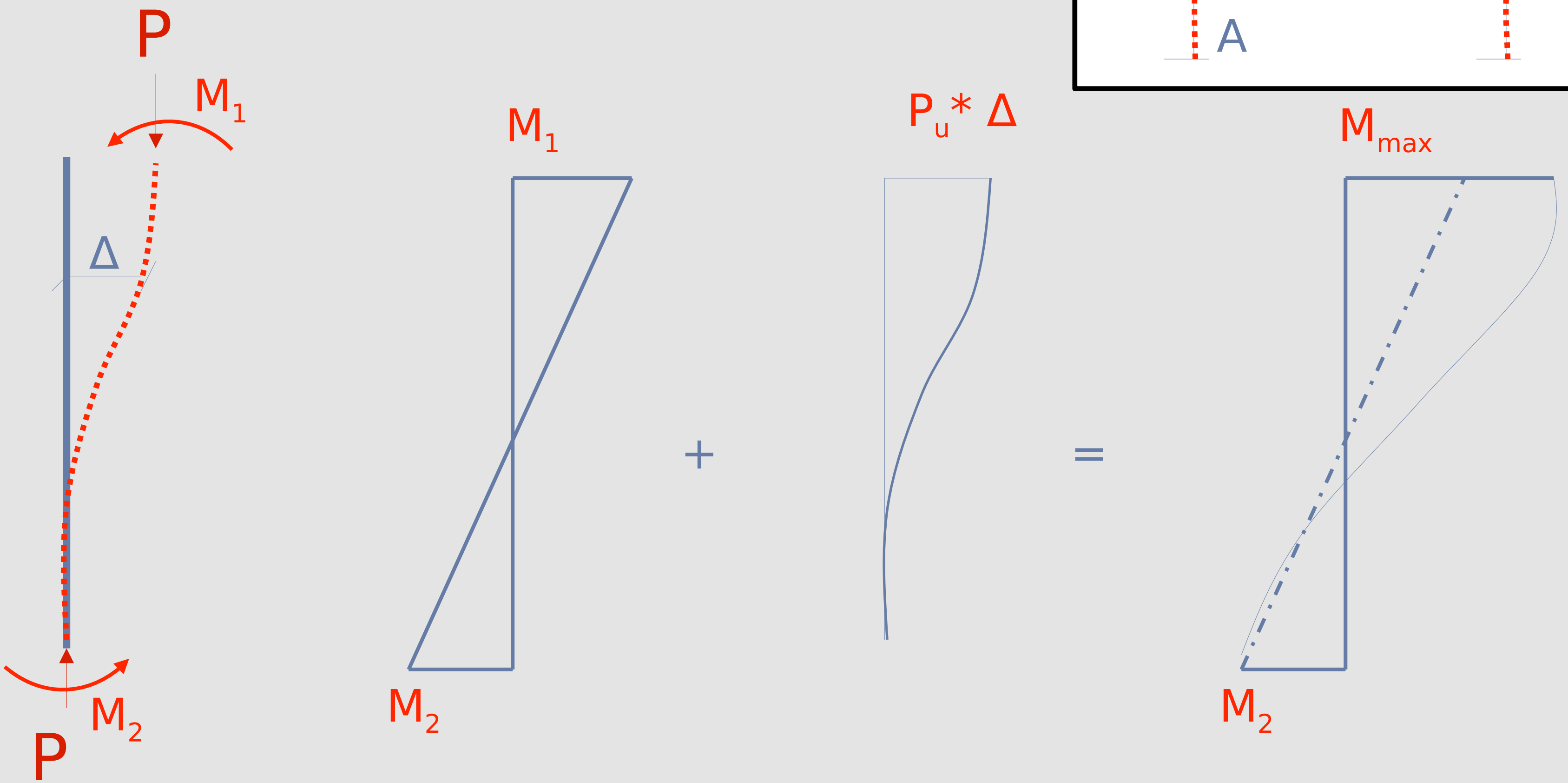
1st Order Axial Load

B_2 Multiplier for Sidesway

- In beam-columns whose ends can translate relative to each other:
  - Maximum primary moment due to sidesway is almost always at one end.
  - Maximum secondary moment due to sidesway is always at one end.
  - Amount of sidesway impacts the secondary moment, and is dependent on the deflection of a system of members acting together.

B_2 Multiplier for Sidesway





B_2 Multiplier for Sidesway

- Because 1st Order and 2nd Order effects are always additive for sidesway, there is no C_m term in the expression for B_2.

B_2 Multiplier for Sidesway

- $B_2 = 1/(1-\alpha\Sigma P_{nt}/\Sigma P_{e2})$ where
  - $\alpha = 1.00$ (LRFD)
  - ΣP_{nt} = Sum of all factored loads acting on all columns in the story
  - $\Sigma P_{e2} = \Sigma\{(\pi EI)/(K_2 L)^2\}$ or $R_M(\Sigma HL/\Delta H)$

- See AISC 360-10 Appendix 8 for all Equations

Braced vs. Unbraced Frames

- The two different moments need to be split and amplified separately.
- Calculate amplified loads as follows:
  - $M_r = B_1 M_{nt} + B_2 M_{lt}$ (AISC 360-10 Equation A-8-1)
  - $P_r = P_{nt} + B_2 P_{lt}$ (AISC 360-10 Equation A-8-2)
    - B_1 = amplification factor for no lateral translation
    - M_{nt} = maximum moment assuming no lateral translation (calculated regardless of braced vs. unbraced)
    - B_2 = amplification factor for lateral translation
    - M_{lt} = maximum moment caused by lateral translation (0 if part of a braced frame)

Interaction Equations

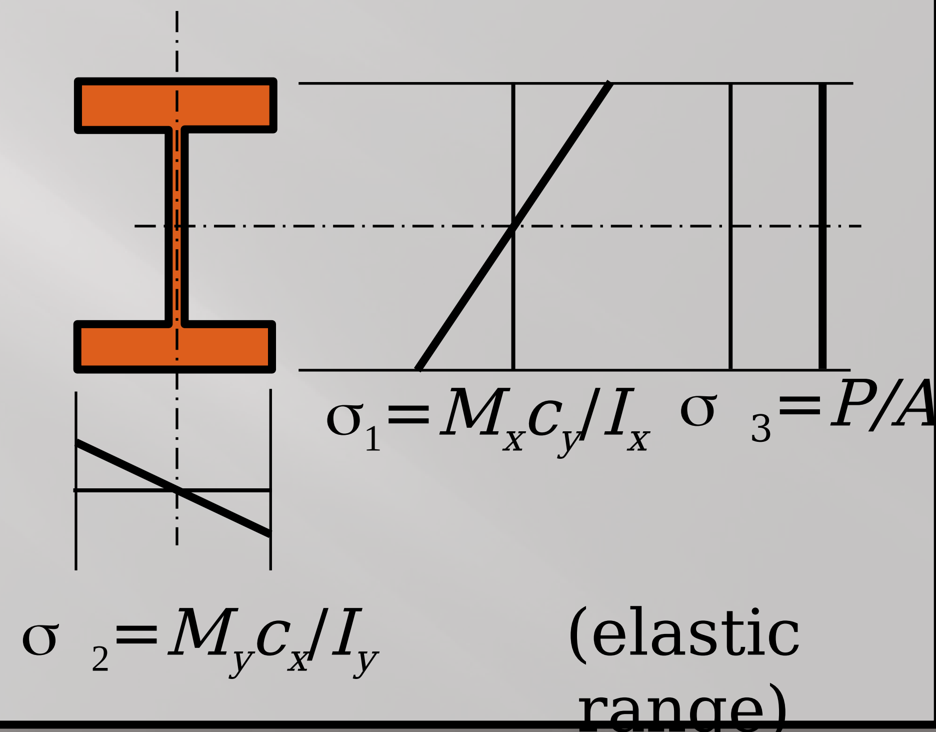
Interaction Equations

- Each part of a member cross-section has a certain capacity.
- Bending loads “use up” some of the capacity.
- Axial loads “use up” some of the capacity.
- The effects of both bending and axial loads need to be combined in some way.

Combination of multiple states of stress:

Bending about the major and minor axis will combine to provide maximum stresses in the corner of a W shape.

Axial load will provide uniform stresses across the member and add to other maximum stresses.



Basic Interaction Equations

- Recall the Basic Design Relationship:
  - Demand ≤ Capacity

    or
  - Demand/Capacity ≤ 1.0

- If there is more than one demand to capacity relationship:
  - Σ(Demand / Capacity) ≤ 1.0

Basic Interaction Equations

- For the case of combined bending and axial load:
  - Columns $P_u \leq \Phi_c P_n$ or D/C = $P_u/\Phi_c P_n$
  - Beams $M_u \leq \Phi_b M_n$ or D/C = $M_u/\Phi_b M_n$
- Basic interaction formula if bending is about 1 axis:
  - $P_u/\Phi_c P_n + M_u/\Phi_b M_n \leq 1.0$
- Basic interaction formula if bending is about 2 axes:
  - $P_u/\Phi_c P_n + M_{ux}/\Phi_b M_{nx} + M_{uy}/\Phi_b M_{ny} \leq 1.0$

AISC Interaction Equations

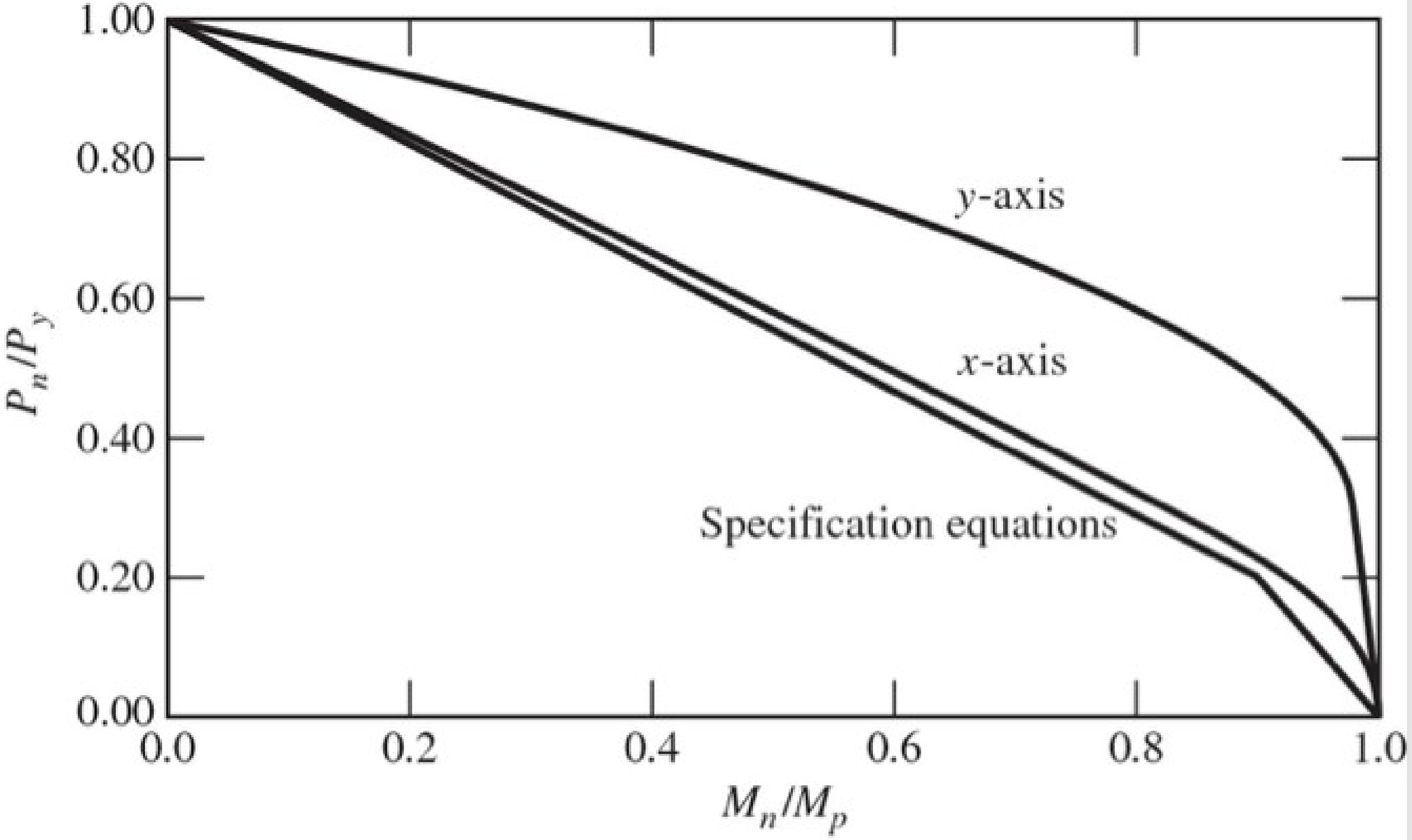


Figure 8.4b

AISC Interaction Equations

- For $P_r/P_c \geq 0.2$
  - $P_r/P_c + (8/9)(M_{rx}/M_{cx} + M_{ry}/M_{cy}) \leq 1.0$
  - AISC 360-10 Equation H1-1a (pg 16.1-73)
  - If axial demand is large, the bending term is slightly reduced.
- For $P_r/P_c < 0.2$
  - $P_r/2P_c + M_{rx}/M_{cx} + M_{ry}/M_{cy} \leq 1.0$
  - AISC 360-10 Equation H1-1b (pg 16.1-73)
  - If axial demand is small, the axial term is reduced.
- Based on matching experimental data

AISC Interaction Equations for Tension

- P_r = required axial tensile strength
  - Using factored loads
- P_c = available axial tensile strength (capacity)

  $\phi_t P_n$ where ϕ_t = 0.9 or 0.75 per Chapter D2
- M_r = required flexural strength
  - Using factored loads
- M_c = available flexural strength (capacity)

  $\phi_b M_n$ where ϕ_b = 0.9

AISC Interaction Equations for Compression

- P_r = required axial compressive strength
  - Using factored loads
- P_c = available axial compressive strength (capacity)
  - $\phi_c P_n$ where $\phi_c = 0.9$
  - Use largest slenderness ratio (KL/r) for either axis
- M_r = required flexural strength
  - Using factored loads
- M_c = available flexural strength (capacity)
  - $\phi_b M_n$ where $\phi_b = 0.9$

Combined Bending & Tension

- For combined compression and flexure
  - If demand/capacity ratios are exceeded then the member is failing in compression in the compression zone of the member.
- For combined tension and flexure
  - If demand/capacity ratios are exceeded then the member is failing in tension in the tension zone of the member.

AISC Design Tables

Table 6-1

Combined Flexure and Compression

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined above.

When $pP_r \geq 0.2$:

$$pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \tag{6-1}$$

When $pP_r < 0.2$:

$$\tfrac{1}{2} pP_r + \tfrac{9}{8}\left(b_x M_{rx} + b_y M_{ry}\right) \leq 1.0 \tag{6-2}$$

Combined Flexure and Tension

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined above.

When $pP_r \geq 0.2$:

$$(t_y \text{ or } t_r)\, P_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \tag{6-3}$$

When $pP_r < 0.2$:

$$\tfrac{1}{2}(t_y \text{ or } t_r)\, P_r + \tfrac{9}{8}(b_x M_{rx} + b_y M_{ry}) \leq 1.0 \tag{6-4}$$

Determination of b_x when C_b > 1.0

The tabulated values of b_x assume that $C_b = 1.0$. These values may be modified in accordance with AISC *Specification* Sections F1 and H1.2. The following procedure may be used to account for $C_b > 1.0$.

$$b_{x(C_b > 1.0)} = \frac{b_{x(C_b = 1.0)}}{C_b} \geq b_{xmin} \tag{6-5}$$

Table 6-1

	LRFD
Axial Compression	$p = \dfrac{1}{\phi_c P_n}, \text{(kips)}^{-1}$
Strong Axis Bending	$b_x = \dfrac{8}{9\phi_b M_{nx}}, \text{(kip-ft)}^{-1}$
Weak Axis Bending	$b_y = \dfrac{8}{9\phi_b M_{ny}}, \text{(kip-ft)}^{-1}$
Tension Yielding	$t_y = \dfrac{1}{\phi_t F_y A_g}, \text{(kips)}^{-1}$
Tension Rupture	$t_r = \dfrac{1}{\phi_t F_u (0.75 A_g)}, \text{(kips)}^{-1}$

AISC Pg 6-3

F_y = 50 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W24

Shape		W24×											
		103[c]				94[c]				84[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.13	0.753	1.27	0.847	1.26	0.840	1.40	0.933	1.46	0.968	1.59	1.06
	11	1.52	1.01	1.42	0.944	1.67	1.11	1.57	1.05	1.92	1.28	1.80	1.20
	12	1.62	1.08	1.46	0.972	1.78	1.18	1.62	1.08	2.03	1.35	1.87	1.24
	13	1.73	1.15	1.51	1.00	1.90	1.26	1.68	1.12	2.17	1.44	1.93	1.28
	14	1.86	1.23	1.55	1.03	2.04	1.36	1.73	1.15	2.33	1.55	2.00	1.33
	15	2.00	1.33	1.61	1.07	2.21	1.47	1.79	1.19	2.52	1.68	2.08	1.38
	16	2.18	1.45	1.66	1.10	2.40	1.60	1.86	1.24	2.75	1.83	2.16	1.44
	17	2.38	1.58	1.72	1.14	2.62	1.74	1.93	1.28	3.01	2.00	2.25	1.49
	18	2.61	1.74	1.78	1.19	2.88	1.92	2.01	1.33	3.32	2.21	2.34	1.56
	19	2.88	1.92	1.85	1.23	3.18	2.12	2.09	1.39	3.68	2.45	2.45	1.63
	20	3.19	2.12	1.92	1.28	3.53	2.35	2.17	1.45	4.08	2.71	2.56	1.70
	22	3.86	2.57	2.09	1.39	4.27	2.84	2.43	1.61	4.94	3.28	2.95	1.96
	24	4.60	3.06	2.37	1.58	5.08	3.38	2.76	1.84	5.88	3.91	3.37	2.24
	26	5.40	3.59	2.65	1.77	5.96	3.97	3.10	2.06	6.90	4.59	3.80	2.53
	28	6.26	4.16	2.94	1.95	6.92	4.60	3.44	2.29	8.00	5.32	4.24	2.82
	30	7.19	4.78	3.22	2.14	7.94	5.28	3.79	2.52	9.18	6.11	4.67	3.11
	32	8.18	5.44	3.50	2.33	9.03	6.01	4.13	2.75	10.4	6.95	5.11	3.40

Other Constants and Properties

	103 ASD	103 LRFD	94 ASD	94 LRFD	84 ASD	84 LRFD
$b_y \times 10^3$, (kip-ft)$^{-1}$	8.58	5.71	9.50	6.32	10.9	7.27
$t_y \times 10^3$, (kips)$^{-1}$	1.10	0.733	1.21	0.802	1.35	0.900
$t_r \times 10^3$, (kips)$^{-1}$	1.35	0.903	1.48	0.987	1.66	1.11
r_x/r_y	5.03		4.98		5.02	
r_y, in.	1.99		1.98		1.95	

[c] Shape is slender for compression with F_y = 50 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending

Shape	W24×											
	103[c]				94[c]				84[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1.13	0.753	1.27	0.847	1.26	0.840	1.40	0.933	1.46	0.968	1.59	1.06
11	1.52	1.01	1.42	0.944	1.67	1.11	1.57	1.05	1.92	1.28	1.80	1.20
12	1.62	1.08	1.46	0.972	1.78	1.18	1.62	1.08	2.03	1.35	1.87	1.24
13	1.73	1.15	1.51	1.00	1.90	1.26	1.68	1.12	2.17	1.44	1.93	1.28
14	1.86	1.23	1.55	1.03	2.04	1.36	1.73	1.15	2.33	1.55	2.00	1.33
15	2.00	1.33	1.61	1.07	2.21	1.47	1.79	1.19	2.52	1.68	2.08	1.38
16	2.18	1.45	1.66	1.10	2.40	1.60	1.86	1.24	2.75	1.83	2.16	1.44
17	2.38	1.58	1.72	1.14	2.62	1.74	1.93	1.28	3.01	2.00	2.25	1.49
18	2.61	1.74	1.78	1.19	2.88	1.92	2.01	1.33	3.32	2.21	2.34	1.56
19	2.88	1.92	1.85	1.23	3.18	2.12	2.09	1.39	3.68	2.45	2.45	1.63
20	3.19	2.12	1.92	1.28	3.53	2.35	2.17	1.45	4.08	2.71	2.56	1.70

(Row labels: least radius of gyration, r_y, X-X axis bending)

Other Constants and Properties

	103 ASD	103 LRFD	94 ASD	94 LRFD	84 ASD	84 LRFD
$b_y \times 10^3$, (kip-ft)$^{-1}$	8.58	5.71	9.50	6.32	10.9	7.27
$t_y \times 10^3$, (kips)$^{-1}$	1.10	0.733	1.21	0.802	1.35	0.900
$t_r \times 10^3$, (kips)$^{-1}$	1.35	0.903	1.48	0.987	1.66	1.11
r_x/r_y	5.03		4.98		5.02	
r_y, in.	1.99		1.98		1.95	

[c] Shape is slender for compression with $F_y = 50$ ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Questions